- Prove that, for $f(z) = \frac{az+b}{cz+d}$, $a, b, c, d \in \mathbb{R}$ and $ad - bc = 1$, we have

$$\frac{1}{\Im z} = \frac{|f'(z)|}{\Im f(z)} \qquad \text{where} \qquad \Im(x + yi) = y$$

  Therefore, PSL(2, $\mathbb{R}$) acts by isometries on the upper-half plane $\mathbb{H}^2$.

- Show that PSL(2, $\mathbb{R}$) is exactly the group of orientation preserving isometries of $\mathbb{H}^2$. Hint: Show that, for every $d > 0$ and every two points $x, y \in H^2$ whose distance is d there is a linear fractional transformation that sends x to i and y to $e^d i$. Then show that every orientation preserving isometry that fixes two points in $\mathbb{H}^2$ is the identity.

- Prove that the universal cover of every complete hyperbolic surface X is isometric to $\mathbb{H}^2$. Here are some steps:

  - Fix a point $x \in X$ and a chart (U_0, ϕ_0) around x. Every path in X connecting x to y can be covered by local charts $(U_0, \phi_0), \ldots, (U_k, \phi_k)$. To this path we associate the point
  $$\left(\prod_{i=0}^{k-1} (\phi_i \phi_{i+1}^{-1})\right) \phi_k(y)$$
    Note that $(\phi_i \phi_{i+1}^{-1})$ is a global isometry of $\mathbb{H}^2$.
  - This association does not depend on the charts used to cover the path or on the homotopy class of the path. Hence it defines a map from $\tilde{X}$ to $\mathbb{H}^2$.
  - The map is 1-1 since homotopy between arcs in $\mathbb{H}^2$ can be pulled back to homotopy between arcs in X.
  - The map in onto since the universal cover is complete.

- Prove that the lift of an essential curve on X to $\mathbb{H}^2$ has two end points that are the fixed points of a loxodromic element in PSL(2, $\mathbb{R}$). Therefore, every curve on X has a unique geodesic representative.

- Prove that for every 3 positive real number $a, b, c > 0$, there exists a unique right-angled hyperbolic hexagon where the length of alternative edges are given by a, b and c. Use this to show that every pair of pants has a reflection symmetry and is a union of 2 identical hexagons.

- Let S be a surface of genus g with b boundary components. Then the number of curves in any pants decomposition is $3g - 3 + b$. Therefore, the Teichmüller space of complete hyperbolic metrics of finite area on the interior of S has dimension $6g - 6 + 2b$.