Hilbert's twelfth problem and *p*-adic variations of modular forms

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Cyclotomic fields are extensions of $\mathbb{Q}$ obtained by adjoining roots of unity; the latter can be viewed as values of the exponential function $e^{2\pi iz}$ at the rationals. The Kronecker-Weber Theorem asserts that every finite abelian extension of $\mathbb{Q}$ is contained in a cyclotomic extension. What is the analogue of this statement for number fields? In his Twelfth Problem, Hilbert asked for explicit analytic formulas for generators of abelian extensions of number fields. In the case of imaginary quadratic fields, an answer can be found in the theory of complex multiplication for elliptic curves but the general case is much more mysterious. In this talk, I will discuss *p*-adic approaches to Hilbert's Twelfth problem for real quadratic fields, relying on the study of *p*-adic variations of modular forms and Galois representations.